

# A Level Further Mathematics Introduction and Summer Work

## Introduction

Remember that A level Further Mathematics is an additional A level to Maths. You will need to bring the same equipment to lessons, although you'll need to have a separate additional folder and notepads for further maths. The content of the course is very interlinked with maths but from an organisation standpoint, must be treated separately.

## Summer Work

This summer work problem set has been designed to challenge your resilience and problem solving skills. We are not expecting you to get everything correct and neither does your outcome here dictate your entry onto the course. The questions are challenging, but are based on GCSE knowledge as well as some problem solving questions akin to UKMT.

What we are looking for is how well you can communicate mathematically. A list of correct answers will score you some marks, but **well explained**, **coherent** and **concise** solutions will score you many more marks.

You must write your solutions neatly onto lined (or squared) A4 paper and bring them to your first Further Maths lesson in September.

If you have any questions please contact either of us on the email address below. Best of luck with your exam results and we look forward to welcoming you in September.

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## Section 1 – Algebraic reasoning

- Expand and simplify  $(x + y)^4$
- Given that  $x$  is positive, solve the equation

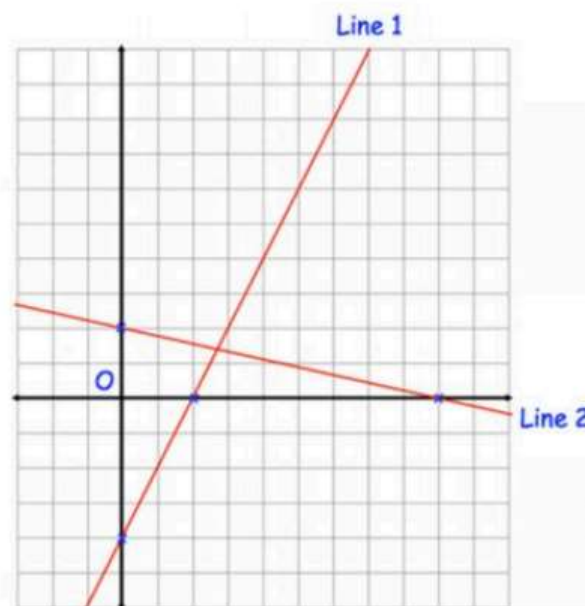
$$\frac{1}{x} + \frac{1}{x+2} = \frac{28}{195}$$

- A straight line passes through  $(7a, 5)$  and  $(3a, 3)$ .  
An equation of this line is  $x + by - 12 = 0$ . Find the values of  $a$  and  $b$ .
- Ben is using the quadratic formula to solve an equation of the form  $ax^2 + bx + c = 0$ . He has got to this point in his working out:

$$x = \frac{3 \pm \sqrt{29}}{2}$$

Find the values of  $a$ ,  $b$  and  $c$  for the equation he is solving.

- Shown are two straight lines drawn on the grid. Line 1 has equation  $y = 3x - 12$ .  
(a) Find the equation of line 2.  
(b) Are the lines perpendicular? Explain your answer.



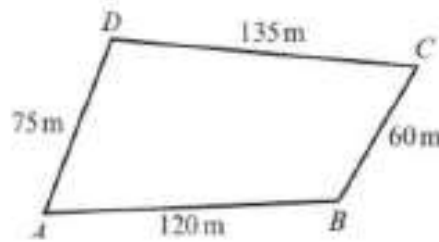
6. The graph of  $y = ax^2 + bx + c$  has a vertex at  $(5, -3)$  and the curve passes through  $(4, 0)$ . Find the values of  $a$ ,  $b$  and  $c$ .
7. A swimmer dives into a pool. Her position,  $p$  metres, underwater can be modelled in relation to her horizontal distance,  $x$  metres, from the point she entered the water as a quadratic equation  $p = \frac{1}{2}x^2 - 3x$ . The position of the bottom of the pool can be modelled by the linear equation  $p = 0.3x - 6$ . Determine whether this model predicts that the swimmer will touch the bottom of the pool.
8. Accident investigators are studying the stopping distance of a particular car.  
 When the car is travelling at 20mph, its stopping distance is 6 feet.  
 When the car is travelling at 30mph, its stopping distance is 14 feet.  
 When the car is travelling at 40mph, its stopping distance is 24 feet.  
 The investigators suggest that the stopping distance,  $D$  (in feet) can be modelled by a quadratic function in terms of speed,  $s$  (in miles per hour), as

$$D(s) = as^2 + bs + c$$

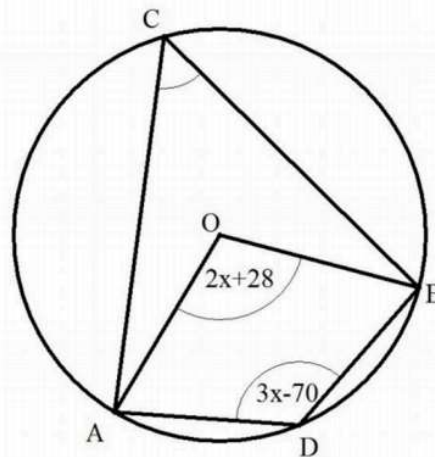
- (a) Find the values of the constants  $a$ ,  $b$  and  $c$ .
- (b) At an accident scene a car has left behind a skid that is 20 feet long. Use the model to calculate the speed that this car was going before the accident.
9. By first completing the square on the equation  $ax^2 + 2bx + c = 0$ ,
- a) Show that:  $\left(x + \frac{b}{a}\right)^2 = \frac{b^2}{a^2} - \frac{c}{a}$
- b) Hence solve  $ax^2 + 2bx + c = 0$  simplifying your answer as much as possible

**Section 2 – Geometrical reasoning**

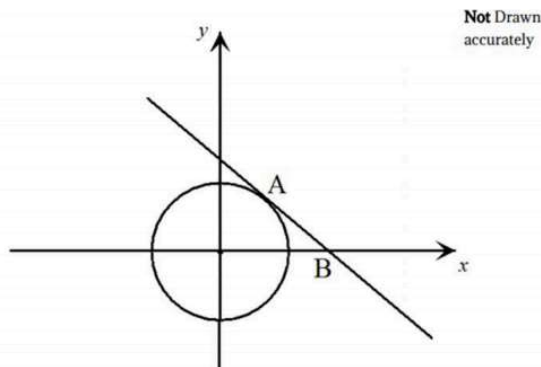
1. A farmer has a field in the shape of a quadrilateral as shown. The angle between fences  $AB$  and  $AD$  is  $74^\circ$ . Find the angle between fences  $BC$  and  $CD$ .



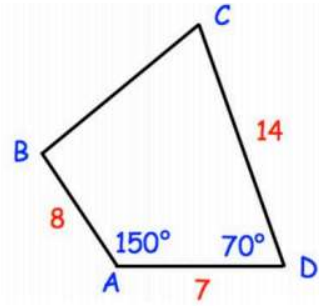
2. Points  $A$ ,  $B$ ,  $C$  and  $D$  are all on the circumference of the circle.  $O$  represents the centre. Angles  $AOB$  and  $ADB$  are expressed in terms of  $x$ . Calculate the value of  $x$ , giving a reason for each stage of your working.



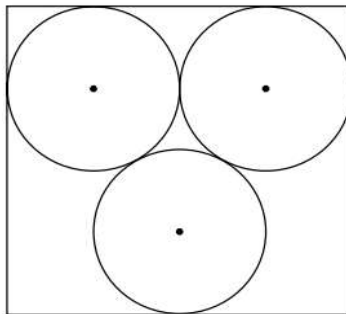
3. Consider the circle with equation  $x^2 + y^2 = 13$  sketched below. The point  $A$  lies on the circle and has a  $y$ -ordinate of 2. The tangent to the circle at  $A$  intersects the  $x$ -axis at the point  $B$ . Find the coordinates of  $B$ .



4. In quadrilateral ABCD, various lengths and angles have been labelled. Calculate the length of BC. Given your answer correct to 3 significant figures.



5. The diagram shows 3 identical circles inside a rectangle. Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram. Given that the radius of each circle is 24mm, work out the area of the rectangle. Give your answer correct to 3 significant figures.



**Section 3 – Number (these questions require rigorous reasoning and justification)**

1. Which of the following cannot be a remainder when a square number is divided by 6:      0    1    2    3    4
  
2. Three consecutive even numbers are such that the sum of four times the smallest and twice the largest exceeds three times the second by 2006. What is the sum of the digits of the smallest number?
  
3. The positive integer 'n' is between 1 and 20. Lane adds up the integers from 1 to n (including 1 and n). Melvin adds up all the integers from n+1 to 20 (including n+1 and 20). Their totals are the same. What is the value of n?
  
4. The digits from 1 to 9 are to be written in the nine cells in the 3x3 grid shown below with one digit per cell.

The product of the three digits in the first row is 12

The product of the three digits in the second row is 112

The product of the three digits in the first column is 216

The product of the three digits in the second column is 12

What is the product of the digits in the shaded cells?

<b>216</b>	<b>12</b>		
			<b>12</b>
			<b>112</b>